CHAPTER 6

SETS AND SUBSETS

Since sets, subsets, and Boolean algebra satisfy the same laws (that is, have similar properties), we will discuss sets and subsets as a means of introducing Boolean algebra. It should be understood, though, that sets and subsets are related to all other branches of mathematics.

SETS

The meaning of a set is any well-defined collection, group, list, or class of objects which possess (or do not possess) some common property whereby we can determine membership in that set. The object or element may take any form such as articles, people, conditions, or numbers. Membership is the fundamental relation of set theory.

NOTATION

When appropriate we will use capital letters to designate sets and lower case letters to indicate elements of a set. Generally, sets are designated by the following:

1. Description.
2. Tabulation.
3. Capital letters.
4. Set builder notation.
5. Combinations of the above.

Examples of each form of notation, in order, are as follows:

1. "the set of odd prime numbers less than ten."
2. $\{7, 3, 5\}$ or $\{0, 2, 4, 6, \cdots\}$
3. A, B, C, $\cdots$
4. $\{x | x$ is a natural number$\}$ or $\{x | x = N\}$
5. $A = \{x | x^2 = 4\}$

In item 4 we let | mean "such that" and x represents any element of the set. Also, the x to the left of | is a variable and the defining property which is required to belong to the set is to the right of | .

Item 5 is read as "A is the set of numbers x such that x square equals four."

Examples of sets and their designations are as follows:

1. The numbers 3, 5, 7, and 11.
   $\{3, 5, 7, 11\}$
2. The letters of the alphabet between c and i.
   $\{d, e, f, g, h\}$
3. Members of the Navy.
   $\{x | x$ is a member of the Navy$\}$
4. Solutions of the equation $x^2 + 3x - 4 = 0$.
   $\{x | x^2 + 3x - 4 = 0\}$

If set A contains x as one of its elements, we indicate this membership by writing

$$x \in A$$

and if set A does not contain x, we write

$$x \notin A$$

That is, if

$$A = \{2, 4, 6, 8, \cdots\}$$

then

$$2 \in A,\ 3 \notin A,\ 13 \notin A,\ 12 \in A, \text{ etc.}$$

FINITE AND INFINITE SETS

We define a finite set as one in which its elements or members could be counted; that is, the counting process or enumeration of its elements would at some time come to an end. This count is called the cardinal number of the set. An infinite set is one which is not finite.

The following list of examples illustrates the distinction between finite and infinite sets:

1. If A is the set of days in the month of December, then set A is finite.
2. If set $B = \{1, 3, 5, 7, \cdots\}$, then set B is infinite.

3. If set $C = \{x \mid x$ is a grain of sand on the earth$\}$, then set C is finite. (The counting process would be difficult but would come to an end.)

4. If set $D = \{x \mid x$ is an animal on the earth$\}$, then set D is finite.

EQUALITY OF SETS

Sets A and B are said to be equal if and only if every element of A is an element of B and every element of B is an element of A. In this case we write

$$A = B$$

If

$$A = \{2, 3, 4, 5\}$$

and

$$B = \{5, 3, 4, 2\}$$

then

$$A = B$$

Notice that each of the elements 2, 3, 4, and 5 in A is in B and each of the elements in B is in A although the order of elements is different. The arrangement of elements does not change the set. Also, the set does not change if some elements are repeated.

If

$$A = \{3, 9, 9, 7\}$$

and

$$B = \{3, 3, 9, 7\}$$

then

$$A = B$$

In some cases the equality of sets is not obvious, as shown in the following:

If

$$A = \{x \mid x^2 - 5x = -6\}$$

and

$$B = \{2, 3\}$$

and

$$C = \{x \mid 1 < x < 4,\ x \text{ an integer}\}$$

then

$$A = B = C$$

NULL SET

We define the null set as a set which has no members or elements. This is a set which is void or empty. We denote this set by the symbol $\emptyset$ or $\{\ \}$. Notice that $\emptyset$ is not the same as 0; that is, 0 is not a set but $\emptyset$ is. Also, $\emptyset$ is not the same as $\{0\}$ because $\emptyset$ is the null set and $\{0\}$ is a set with the one element 0. The empty or null set may also be defined by a statement which prohibits any element from being a member of the set; that is, the set exists but has no members.

This is similar to regarding zero as a number; that is, the natural numbers are 1, 2, 3, ··· and are used for counting, and zero is not a counting number but it is used to indicate that there is nothing to count.

There is only one empty or null set because two sets are equal if they consist of the same elements, and since the empty sets have no members they are equal.

If

$$A = \{x \mid x \text{ is a 300-year-old man on earth}\}$$

then, as far as we know, A is the null set.

If

$$B = \{x \mid x^2 = 9,\ x \text{ is even}\}$$

then

$$B = \text{the null set}$$

or

$$B = \emptyset \quad \text{or} \quad \{\ \}$$

PROBLEMS:

1. Use set notation to rewrite the following statements:
   a. x does not belong to C.
   b. k is a member of B.

2. Write the following, using set-builder notation.
   a. $B = \{2, 4, 6, 8, \cdots\}$.
   b. C is the set of men in the Navy

3. Indicate which sets are finite.
   a. The days of the week.
   b. $\{x | x \text{ is an odd integer}\}$.
   c. $\{3, 6, 9, \cdots\}$.

4. Which pairs of sets are equal?
   a. $\{1, 2, 3\}$ and $\{2, 1, 3, 2\}$
   b. $\{k, 1, x\}$ and $\{x, k, n, 1\}$

5. Which of the following describe the null set?
   a. $C = \{x | x + 6 = 6\}$.
   b. $B = \{x | x$ is a positive integer less than one$\}$.

ANSWERS:

1. a. $x \notin C$
   b. $k \in B$

2. a. $B = \{x | x \text{ is even}\}$
   b. $C = \{x | x \text{ is a man in the Navy}\}$

3. a. finite
   b. infinite
   c. infinite

4. a. equal
   b. unequal

5. a. not the null set
   b. the null set

SUBSETS

We say that set B is a subset of set A if and only if every element of set B is an element of set A.

If we have the situation where

$$A = \{1, 2, 3\}$$

and

$$B = \{1, 2\}$$

then B is a subset of A and we write

$$B \subset A$$

Notice that every element of B is an element of A; that is,

$$x \in B \rightarrow x \in A$$

where the symbol $\rightarrow$ means "implies" or if the first $(x \in B)$ is true then the second $(x \in A)$ is true. Also, it should be noted that the null set is a subset of every set.

If

$$D = \{x | x \text{ is an odd integer}\}$$

and

$$E = \{1, 3, 5, 7\}$$

then

$$E \subset D$$

PROPER SUBSET

If we have two sets such that

$$F = G$$

we may write

$$F \subseteq G$$

and

$$G \subset F$$

and we say F is a subset of G and G is a subset of F.

If we write

$$K = K$$

we say K is a subset of itself.

Since there is a distinction between these subsets and the subsets previously mentioned, we may call the previous subsets "proper subsets"; that is, B is a proper subset of A if B is a subset of A and at the same time B is not equal to A.

When we have

$$C = \{3, 4, 5\}$$

and

$$D = \{3, 4, 5\}$$

then C is a subset of D and we properly write

$$C \subseteq D$$

which indicates that C is also equal to D.

Although this distinction is made in some studies of sets and subsets, we will not make any distinction in the following discussions.

COMPARABILITY

If we have two sets where A is a subset of B or B is a subset of A, then we say sets A and B are comparable; that is, if

$$A \subset B$$

or

$$B \subset A$$

then A and B are comparable.

For two sets to be noncomparable we must have the relations

$$A \not\subset B$$

and

$$B \not\subset A$$

where the symbol $\not\subset$ means "is not a subset of."

If

$$C = \{3, 5, 6\}$$

and

$$D = \{5, 6\}$$

then C and D are comparable. This is because

$$D \subset C$$

If

$$E = \{7, 8, 9\}$$

and

$$F = \{8, 9, 10\}$$

then E and F are noncomparable. This is because there is an element in E not in F and there is an element in F not in E. This is written

$$7 \in E \quad \text{and} \quad 7 \notin F$$

and

$$10 \notin E \quad \text{and} \quad 10 \in F$$

UNIVERSAL SET

When we investigate sets whose elements are natural numbers, we say the natural numbers comprise the universal set. Generally, we say the universe is the set of natural numbers. We denote the universal set by the letter U.

If we are discussing sets of the letters of our alphabet, we call the alphabet the universe.

If we are talking about humans, then the universal set consists of all people on earth.

POWER SET

If we have a set A such that

$$A = \{1, 2, 4\}$$

and we list each subset of A; that is,

$$\{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}, \{\ \}$$

we call this family of sets the power set of A.

The power set of any set S is the family of all subsets of S. This is denoted by

$$2^S$$

This designation is used because the number of subsets of any finite set of n elements is 2^n. If we have the set B such that

$$B = \{8, 9, 10\}$$

then, the power set of B; that is,

$$2^B$$

has

$$2^n = 2^3$$
$$= 8$$

subsets.

The power set is shown as follows:
If

$$B = \{8, 9, 10\}$$

then

$$2^B = \left\{ B,\ \{8, 9\},\ \{8, 10\},\ \{9, 10\},\ \{8\},\ \{9\},\ \{10\},\ \emptyset \right\}$$

DISJOINT SETS

When we find two sets that have no elements in common, we say these sets are disjoint.
If

$$A = \{1, 2, 3\}$$

and

$$B = \{4, 5, 6\}$$

then A and B are disjoint.
If

$$C = \{a, b, c\}$$

and

$$D = \{c, d, e\}$$

then C and D are not disjoint because

$$c \in C \quad \text{and} \quad c \in D$$

VENN-EULER DIAGRAMS

The use of Venn-Euler diagrams, or simply Venn diagrams, is not an acceptable "proof" of the relationships among sets. Nevertheless we will use these diagrams for our intuitive approach to these relationships. We will use a circle to denote a set and a rectangle to indicate the universe. If we have two sets such that

$$A = \{1, 2, 3\}$$

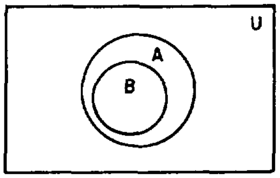
and

$$B = \{2, 3\}$$

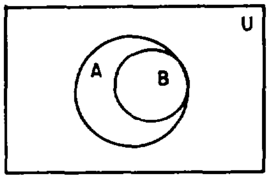
then we may show that

$$B \subset A$$

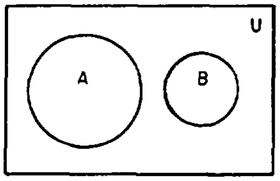
by drawing



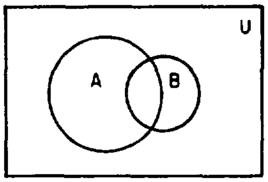
or



If we have two disjoint sets, we draw



If the sets are not disjoint, we draw



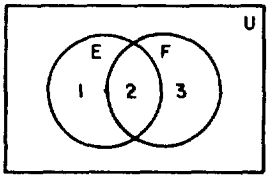
If

$$E = \{1, 2\}$$

and

$$F = \{2, 3\}$$

then we show their relationship by



indicating that

$$1 \in E, \quad 1 \notin F$$
$$2 \in E, \quad 2 \in F$$
$$3 \notin E, \quad 3 \in F$$

PROBLEMS:

1. If set $A = \{2, 3, 7, 9\}$, then how many subsets does A contain?

2. If set $B = \{x | x$ is an integer between 5 and $8\}$, then how many subsets does B contain? Write the power set.

3. Indicate whether the following pairs of sets are comparable or not.

a. $A = \{5, 6, 9\}$ and $B = \{9, 10, 11\}$
b. $C = \{x | x$ is even$\}$ and
$D = \{x | x^2 + 5x = -6\}$
c. $E = \{x | x$ is odd$\}$ and $\emptyset$

4. If

$A = \{1\}$
$B = \{1, 2\}$
$C = \{2, 3, 4\}$
$D = \{3, 4\}$

and

$E = \{1, 3, 4\}$

then indicate whether the following are true or false.

a. $D \subset C$
b. $A \neq E$
c. $A \not\subset D$
d. $A \subset C$
e. $D \not\subset E$
f. $B \subset E$

5. Make a Venn diagram of the following relationships.

a. $C \subset B$, $B \subset A$
b. $C \subset B$, $D \subset B$, $B \subset A$, and C and D are disjoint.
c. $A = \{1, 2, 3\}$, $B = \{2, 4\}$
d. $A = \{1, 2, 3, 4\}$, $B = \{4\}$,
$C = \{3, 4, 5\}$

ANSWERS:

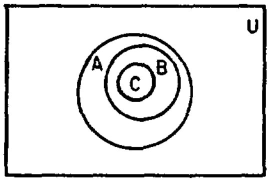
1. 2^4 or 16

2. 2^2 or 4; $\{B, \{6\}, \{7\}, \emptyset\}$

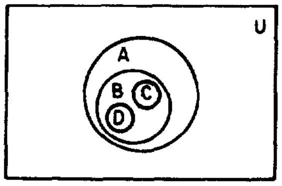
3. a. not comparable
b. not comparable
c. comparable

4. a. true
b. true
c. true
d. false
e. false
f. false

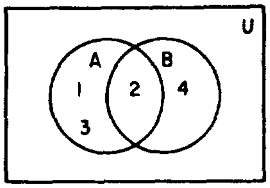
5. a.



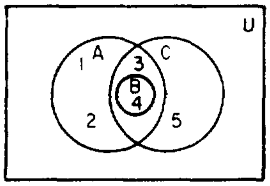
b.



c.



d.



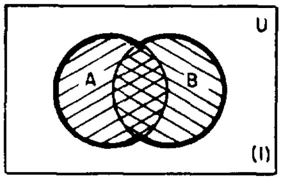
OPERATIONS

When operating with arithmetic, the process of adding a pair of numbers A and B produces a number A + B called the sum. Subtraction of B from A produces a number A - B called the difference and multiplication of A and B produces AB called the product. In this chapter we will discuss operations of sets which are somewhat similar to the arithmetic operations of addition, subtraction, and multiplication. The set operations are union, intersection, and difference. Complements will also be discussed.

The operations of sets will be discussed in relation to Venn diagrams which is a "show" rather than a "prove" type approach. The laws of sets will be discussed later in the chapter.

UNION

We say that the union of two sets A and B is the set of all elements which belong to A or B or to both A and B. We indicate the union of A and B by writing $A \cup B$. To show this union by a Venn diagram we draw diagram (1)



where the circles show the sets A and B and the rectangle indicates the universe U. We shaded set A with positive slope lines and shaded set B with negative slope lines. Any part of the universe which is shaded is "A union B" or $A \cup B$; that is,

$$A \cup B = \{x \mid x \in A \quad \text{or} \quad x \in B\}$$

An example using numerals to show the union of two sets A and B is as follows:

If

$$A = \{1, 2, 3, 4\}$$

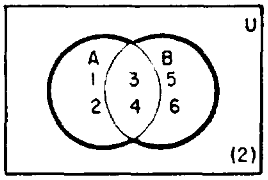
and

$$B = \{3, 4, 5, 6\}$$

then

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

To show this union by Venn diagram, we draw diagram (2)



From the previous discussion it should be apparent that

$$A \cup B = B \cup A$$

In a later discussion we will relate the union of A and B to A + B.

PROBLEMS:

Let $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8\}$, and $C = \{3, 5, 6\}$

Find

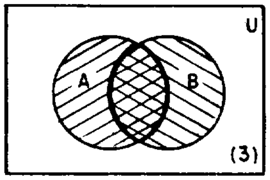
1. $A \cup B$
2. $A \cup C$
3. $B \cup C$
4. $A \cup A$
5. $(A \cup B) \cup C$

ANSWERS:

1. $\{1, 2, 3, 4, 5, 6, 8\}$
2. $\{1, 2, 3, 4, 5, 6\}$
3. $\{2, 3, 4, 5, 6, 8\}$
4. $\{1, 2, 3, 4, 5\}$
5. $\{1, 2, 3, 4, 5, 6, 8\}$

INTERSECTION

We say that the intersection of two sets A and B is the set of all elements which belong to A and B by writing $A \cap B$; that is, $A \cap B = \{x | x \in A, x \in B\}$. To show this intersection by a Venn diagram we draw (3)



A and B have been shaded as before and the intersection of A and B, that is, $A \cap B$ or the area shaded by cross-hatch.

An example using numerals to show the intersection of two sets A and B is as follows:

If

$$A = \{1, 2, 3, 4\}$$

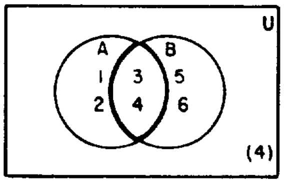
and

$$B = \{3, 4, 5, 6\}$$

then

$$A \cap B = \{3, 4\}$$

To show this intersection by Venn diagram we draw (4)



Again, it should be apparent that

$$A \cap B = B \cap A$$

In a later discussion we will relate the intersection of A and B to $A \cdot B$.

PROBLEMS:

Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 4\}$, and $C = \{3, 4, 5, 6, 7\}$

Find

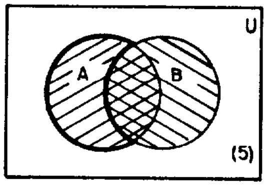
1. $A \cap B$
2. $A \cap C$
3. $B \cap C$
4. $A \cap A$
5. $(A \cap B) \cap C$

ANSWERS:

1. $\{1, 3\}$
2. $\{3, 5\}$
3. $\{3, 4\}$
4. $\{1, 3, 5\}$
5. $\{3\}$

DIFFERENCE

We say that the difference of two sets A and B is the set of elements which belong to A but do not belong to B. We indicate this by writing $A - B$; that is, $A - B = \{x | x \in A, x \notin B\}$. To show this difference by a Venn diagram we draw (5)



A minus B is the area which contains the positive slope shading only.

An example using numerals is as follows: If

$$A = \{1, 2, 3, 4\}$$

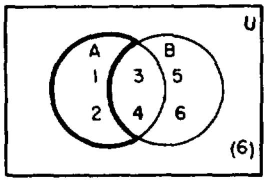
and

$$B = \{3, 4, 5, 6\}$$

then

$$A - B = \{1, 2\}$$

This is shown in Venn diagram form by drawing (6)



PROBLEMS:

Let $A = \{1, 3, 5\}$, $B = \{2, 4, 5\}$, and $C = \{3, 5\}$

Find

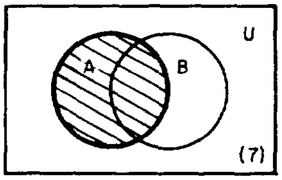
1. A - B
2. A - C
3. B - C
4. B - A
5. A - A

ANSWERS:

1. $\{1, 3\}$
2. $\{1\}$
3. $\{2, 4\}$
4. $\{2, 4\}$
5. $\{\ \}$ or $\emptyset$

COMPLEMENT

We say that the complement of a set A is the set of all elements within the universe which do not belong to A. This is comparable to the universe U minus the set A. We indicate the complement of A by writing $\bar{A}$; that is, $\bar{A} = \{x \mid x \epsilon U, x \notin A\}$. Shown in Venn diagram form this is (7)



The area which is not shaded is the complement of A; that is, $\bar{A}$.

An example using numerals to show the complement of A is as follows: If

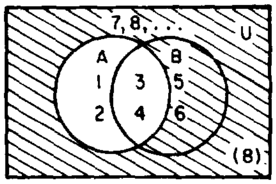
$$A = \{1, 2, 3, 4\} \quad \text{and} \quad B = \{3, 4, 5, 6\}$$

then

$$\bar{A} = \{5, 6, 7, 8, \cdots\}$$

where we assume the universe U to be the set of natural numbers 1, 2, 3, $\cdots$.

Shown in Venn diagram this is (8)



where the shaded area is the complement of A.

If we now use the previous information given, we find that the union of A and its complement $\bar{A}$ is

$$A \cup \bar{A} = U$$

Also, the intersection of A and its complement A is

$$A \cap \bar{A} = \emptyset$$

and the complement of the universal set U is the empty set $\emptyset$; that is,

$$\overline{U} = \emptyset$$

and

$$\overline{\emptyset} = U$$

PROBLEMS:

Let $A = \{2,3,4\}$, $B = \{2,4,6\}$, $C = \{4,5,6\}$, and the universe $U = \{1, 2, 3, \cdots, 10\}$.
Find

1. $\overline{A}$
2. $\overline{B}$
3. $\overline{(A \cup C)}$
4. $\overline{(A \cap B)}$
5. $\overline{(B - C)}$

ANSWERS:

1. $\{1, 5, 6, 7, 8, 9, 10\}$
2. $\{1, 3, 5, 7, 8, 9, 10\}$
3. $\{1, 7, 8, 9, 10\}$
4. $\{1, 3, 5, 6, 7, 8, 9, 10\}$
5. $\{1, 3, 4, 5, 6, 7, 8, 9, 10\}$

LAWS OF ALGEBRA OF SETS

We will discuss the relations that exist between sets and the operations of union, intersection, and complements. These operations satisfy various laws or identities which are called the algebra of sets.

We will use an intuitive approach to understanding these laws and in most cases will show the laws by use of Venn diagrams.

IDEMPOTENT LAWS

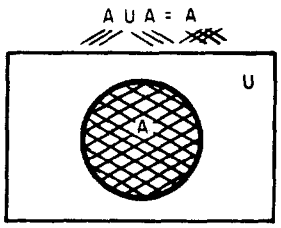
The relations $A \cup A = A$ and $A \cap A = A$ are the idempotent identities. Since the union of two sets A and B is $\{x | x \in A \text{ or } x \in B\}$, it follows that

$$A \cup A = A$$

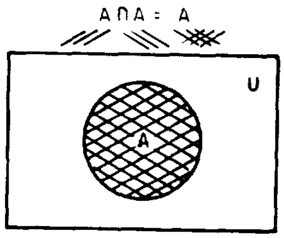
Also, since the intersection of two sets A and B is $\{x | x \in A,\ x \in B\}$, it follows that

$$A \cap A = A$$

Venn diagrams to show this are as follows: (We show each A and how it is shaded.)



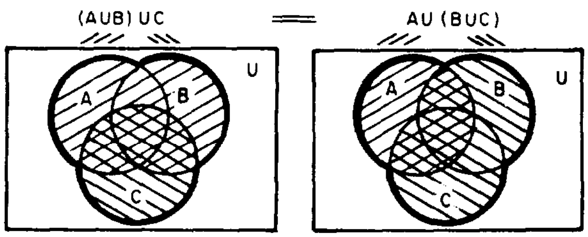
and



In both cases the cross-hatch is the solution area.

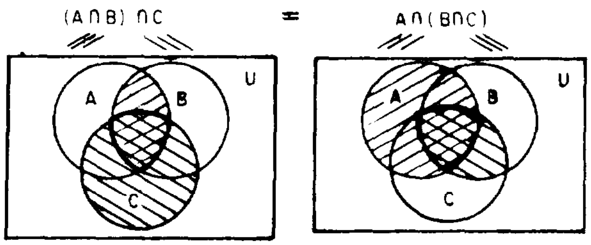
ASSOCIATIVE LAWS

The relations $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$ are the associative identities. We show these as follows:



where the area with any shading is the same in each case.

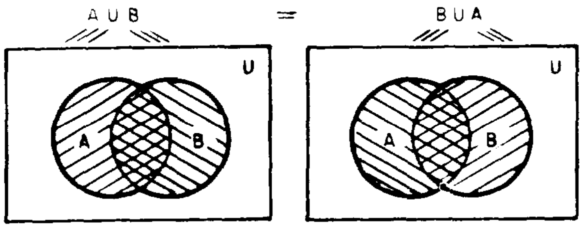
Also,



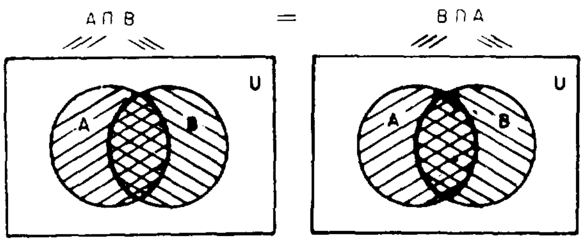
where the cross-hatched areas are the same.

COMMUTATIVE LAWS

The relations $A \cup B = B \cup A$ and $A \cap B = B \cap A$ are the commutative identities. They are shown as follows:

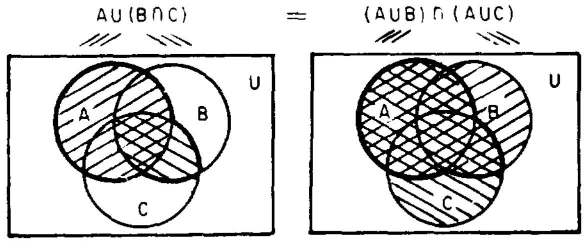


and



DISTRIBUTIVE LAWS

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ are the distributive identities. They indicate that $\cup$ distributes over $\cap$ and that $\cap$ distributes over $\cup$. They are shown as follows:



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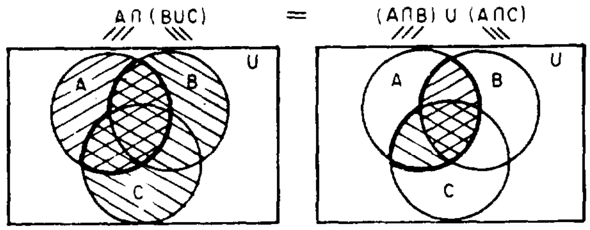
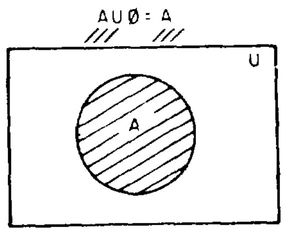
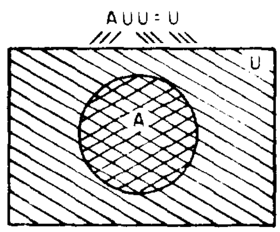


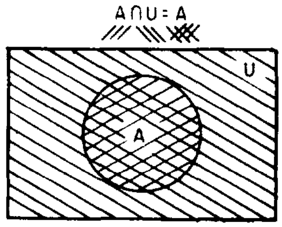
IDENTITY LAWS

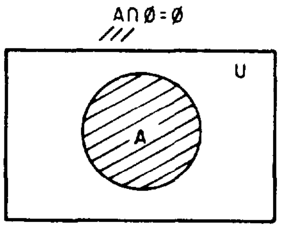
These laws state that $A \cup \emptyset = A$, $A \cup U = U$, $A \cap U = A$, and $A \cap \emptyset = \emptyset$. The Venn diagrams for these are shown as follows:





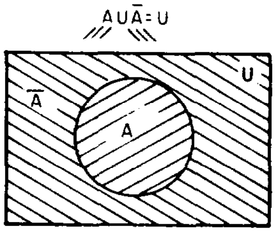
and

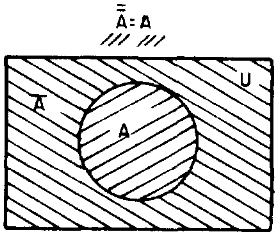


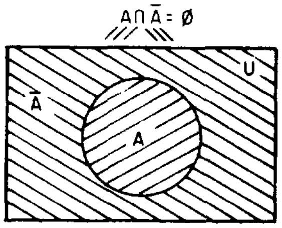


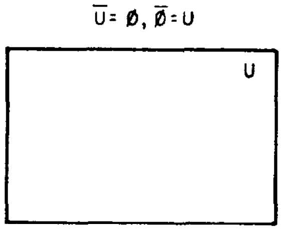
COMPLEMENT LAWS

These laws state that $A \cup \overline{A} = U$, $\overline{\overline{A}} = A$, $A \cap \overline{A} = \emptyset$, and $\overline{U} = \emptyset$, $\overline{\emptyset} = U$. These are shown as follows:



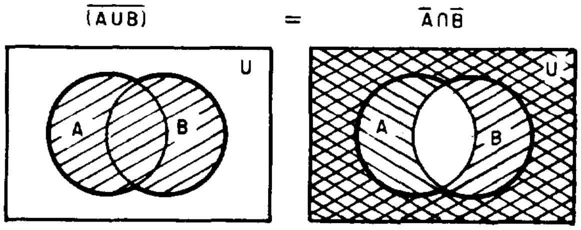




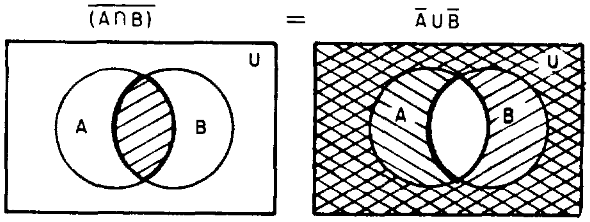


DE MORGAN'S LAWS

DeMorgan's laws are indicated by $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ and $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$. They are shown in Venn diagram form by the following:



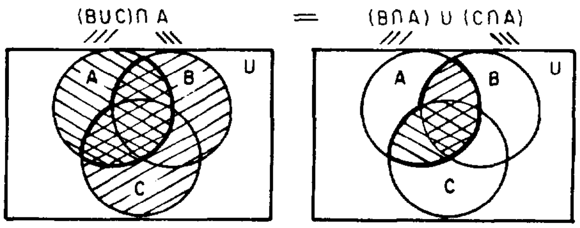
where $\overline{(A \cup B)}$ is the unshaded area and $\overline{A} \cap \overline{B}$ is the crosshatched area. Also,



where $\overline{(A \cap B)}$ is the unshaded area and $\overline{A} \cup \overline{B}$ is the shaded area.

PRINCIPLE OF DUALITY

This principle of the theory of sets states that if we interchange the operations of union and intersection and also interchange the universe and null set in any theorem then the new equation is a valid theorem. We may show this by the following:



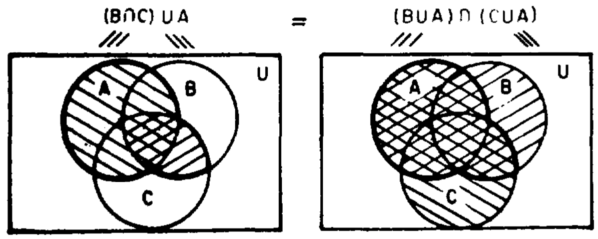
Now, the dual of

$$(B \cup C) \cap A = (B \cap A) \cup (C \cap A) \tag{1}$$

is

$$(B \cap C) \cup A = (B \cup A) \cap (C \cup A) \tag{2}$$

Since we have shown equation (1) to be true, then by the principle of duality equation (2) is true. We may show equation (2) to be true by writing



The following is a summary of the laws of **the algebra** of sets and the principle of duality:

Idempotent Laws

$\mathbf{A} \cup \mathbf{A} = \mathbf{A}$ $\quad$ $A \cap A = A$

Associative Laws

$(\mathbf{A} \cup \mathbf{B}) \cup \mathbf{C} = \mathbf{A} \cup (\mathbf{B} \cup \mathbf{C})$ $\quad$ $(A \cap B) \cap C = A \cap (B \cap C)$

Commutative Laws

$\mathbf{A} \cup \mathbf{B} = \mathbf{B} \cup \mathbf{A}$ $\quad$ $A \cap B = B \cap A$

Distributive Laws

$\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C}) = (\mathbf{A} \cup \mathbf{B}) \cap (\mathbf{A} \cup \mathbf{C})$ $\quad$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Identity Laws

$\mathbf{A} \cup \emptyset = \mathbf{A}$ $\quad$ $A \cap U = A$

$\mathbf{A} \cup \mathbf{U} = \mathbf{U}$ $\quad$ $A \cap \emptyset = \emptyset$

Complement Laws

$\mathbf{A} \cup \bar{\mathbf{A}} = \mathbf{U}$ $\quad$ $A \cap \bar{A} = \emptyset$

$\bar{\bar{\mathbf{A}}} = \mathbf{A}$ $\quad$ $\bar{U} = \emptyset$, $\bar{\emptyset} = U$

DeMorgan's Laws

$\overline{(\mathbf{A} \cup \mathbf{B})} = \bar{\mathbf{A}} \cap \bar{\mathbf{B}}$ $\quad$ $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Principle of Duality

$(\mathbf{B} \cup \mathbf{C}) \cap \mathbf{A} = (B \cap A) \cup (C \cap A)$

$(\mathbf{B} \cap \mathbf{C}) \cup \mathbf{A} = (B \cup A) \cap (C \cup A)$

The following examples illustrate the use of some of the laws of the algebra of sets to change an expression from one form to another. The particular law used in each step is indicated.

EXAMPLE: Prove that

$$C \cap (B \cup A) = (C \cap B) \cup (C \cap A).$$

SOLUTION: Write	Law used
$C \cap (B \cup A) = (C \cap B) \cup (C \cap A)$	distributive

EXAMPLE: Prove that

$$(A \cup B) \cap (B \cup C) = (A \cap C) \cup B.$$

SOLUTION: Write	Law used
$(A \cup B) \cap (B \cup C)$	
$= (B \cup A) \cap (B \cup C)$	commutative
$= B \cup (A \cap C)$	distributive
$= (A \cap C) \cup B$	commutative

EXAMPLE: Prove that

$$(A \cap B) \cup (A \cap \bar{B}) = A.$$

SOLUTION: Write	Law used
$(A \cap B) \cup (A \cap \bar{B})$	
$= A \cap (B \cup \bar{B})$	distributive
$= A \cap U$	complement substitution
$= A$	identity

EXAMPLE: Prove that $A \cup (\bar{A} \cap B) = A \cup B$.

SOLUTION: Write	Law used
$A \cup (\bar{A} \cap B)$	
$= (A \cup \bar{A}) \cap (A \cup B)$	distributive
$= U \cap (A \cup B)$	complement substitution
$= A \cup B$	identity

EXAMPLE: Prove that $A \cup (A \cap B) = A$.

SOLUTION: Write	Law used
$A \cup (A \cap B) = (A \cap U) \cup (A \cap B)$	identity substitution
$= A \cap (U \cup B)$	distributive
$= A \cap U$	identity substitution
$= A$	identity

EXAMPLE: Prove that $A \cap (A \cup B) = A$.

SOLUTION: Write	Law used
$A \cap (A \cup B) = (A \cup \emptyset) \cap (A \cup B)$	identity substitution
$= A \cup (\emptyset \cap B)$	distributive
$= A \cup \emptyset$	identity substitution
$= A$	identity

EXAMPLE: Prove that $(A \cup U) \cap (A \cap \emptyset) = \emptyset$.

SOLUTION: Write	Law used
$(A \cup U) \cap (A \cap \emptyset) = U \cap (A \cap \emptyset)$	identity substitution
$= U \cap \emptyset$	identity substitution
$= \emptyset$	identity

EXAMPLE: Prove that

$$\bar{A} \cup \overline{(B \cup C)} = \overline{(A \cap B)} \cap \overline{(A \cap C)}.$$

SOLUTION: Write	Law used
$\bar{A} \cup \overline{(B \cup C)} = \bar{A} \cup (\bar{B} \cap \bar{C})$	DeMorgan
$= (\bar{A} \cup \bar{B}) \cap (\bar{A} \cup \bar{C})$	distributive
$= \overline{(A \cap B)} \cap \overline{(A \cap C)}$	DeMorgan

EXAMPLE: Prove that $\overline{(\bar{A} \cup B)} = A \cap \bar{B}$.

SOLUTION: Write	Law used
$\overline{(\bar{A} \cup B)} = \bar{\bar{A}} \cap \bar{B}$	DeMorgan
$= A \cap \bar{B}$	complement